# Distributional Effects of Surging Housing Costs under Schwabe's Law

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# Introduction

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  - Wealth inequality: Rising house prices and housing costs affect the wealth distribution

    Summers (2014); Kuhn, Schularick, & Steins (2018); Dustmann, Fitzenberger, & Zimmermann (2018)

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2) How do these relations depend on Schwabe's law?

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$\rightarrow$ Fundamental mechanisms that operate in the absence of incomplete markets

# Two steps of analysis & results

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  - → Rent channel — Schwabe's law dœsn't matter
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- **Step #1: partial equilibrium – analytical analysis**
  - → Rent channel — Schwabe's law dœsn't matter
  - → Amplification of welfare differences — Schwabe's law matters
- **Step #2: general equilibrium – numerical analysis**
  - Policy experiment: abolishing zoning regulations as exogenous event that dampens rent growth
  - → Comovement of rent and wealth inequality
  - → Aggregate welfare effects
  - → Household-specific welfare effects

# Related literature

- **Housing & macro**: Piazzesi & Schneider (2016)
  - Short run: Davis and Heathcote (2005, *IER*); Iacoviello (2005, *AER*); Iacoviello and Neri (2010, *AEJ:M*); Kiyotaki et al. (2011, *JMCB*); Favilukis et al. (2015, *JPE*); Kydland et al. (2016); ...
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- **Saving and wealth inequality**: De Nardi and Fella (2017, *RED*)
  - Most Bewley-Huggett-Aiyagari models study impact of alternative mechanisms on shape of stationary wealth distribution
  - Exceptions (1): Gabaix, Lasry, Lions, and Moll (2016, *Ectra*); Kaymak and Poschke (2016, *JME*); Hubmer, Krusell and Smith (2016)
  - Exceptions (2): Caselli & Ventura (2000, *AER*); Álvarez-Peláez and Díaz (2005, *JME*)

# The model: households

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s.t.

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- Exogenous ex-ante heterogeneity: $W_j(0)$ and $l_j$

# Utility: motivation # 1

- Instantaneous utility $\left(\bar{s} \equiv \sum_j n_j s_j\right)$

$$u(s_j, c_j) = \frac{\left[\left(s_j - \phi\bar{s}\right)^{\theta}\left(c_j\right)^{1-\theta}\right]^{1-\sigma} - 1}{1-\sigma}$$

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*A house may be large or small; as long as the neighboring houses are likewise small, it satisfies all social requirement for a residence.* ***But let there arise next to the little house a palace, and the little house shrinks to a hut*** *[...] the occupant of the relatively little house will always find himself more uncomfortable, more dissatisfied, more cramped within his four walls.*

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- Evidence for status preferences for housing in the US — Bellet (2017)

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⇒ Iff $\phi > 0$, housing expenditure share is declining in income (Schwabe's law)

⇒ Aggregate housing expenditure share is constant over time

$$\overline{e} = \frac{\theta}{1-(1-\theta)\phi}$$

# Results: partial equilibrium

# Wealth inequality: rent channel



**Proposition: Rent channel**

An increase (decrease) in the growth factor of real rents, $\bar{p}(\tau, t)$, contributes to less (more) wealth inequality in period $t$ for $\sigma > 1$.

- The change in the wealth distribution, at any $t$, is described by

$$\frac{\partial \hat{W}_j(t)}{\partial W_j(t)} = \frac{\mu(t)\tilde{w}(t) - w(t)}{W_j(t)^2}$$

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- The propensity to consume

$$\mu(t) = \left[ \int_t^{\infty} \left[ \bar{p}(\tau, t)^{\theta} e^{-\bar{r}(\tau,t) - \frac{\rho}{\sigma - 1}(\tau - t)} \right]^{\frac{\sigma-1}{\sigma}} \mathrm{d}\tau \right]^{-1}$$

where $\bar{r}(\tau, t) \equiv \int_t^{\tau} r(v)\mathrm{d}v$ and $\bar{p}(\tau, t) \equiv \frac{p(\tau)}{p(t)}$ for $\tau \geq t$

▸ Owner vs. renter ▸ $\hat{W}_j$ analytics

# Welfare: price index channel

**Proposition: Welfare**

Welfare of a household $j$ relative to the representative household, at any $t$, is given by

$$\psi_j(t) = \frac{\mathcal{W}_j(t)}{\bar{\mathcal{W}}(t)} \left( \frac{\mathcal{P}_j(t)}{\bar{\mathcal{P}}(t)} \right)^{-1} - 1.$$

- Ideal price index $\mathcal{P}_j(t) = \frac{p(t)^\theta}{\theta^\theta (1-\theta)^{1-\theta}} \frac{1-\theta}{1-e_j}$

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- Price-index channel: two-sectoral structure & non-homothetic preferences

Definition CEV

# Welfare: price index channel (cont')

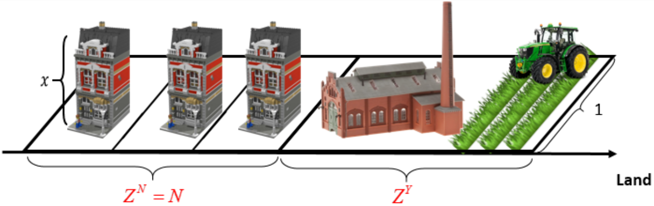
**Corollary: Amplification of welfare differences**

Stronger status concerns amplify, at any $t$, welfare differences, i.e.

$$\frac{\partial \psi_j(t)}{\partial \phi} = \frac{\theta\left[\frac{\mathcal{W}_j(t)}{\overline{\mathcal{W}}(t)}(t) - 1\right]}{(\phi - 1)^2} \begin{cases} > 0 & \text{for } \frac{\mathcal{W}_j(t)}{\overline{\mathcal{W}}(t)} > 1 \\ < 0 & \text{for } \frac{\mathcal{W}_j(t)}{\overline{\mathcal{W}}(t)} < 1 \end{cases}$$

▸ Definition CEV ▸ Analytics

# General equilibrium: production

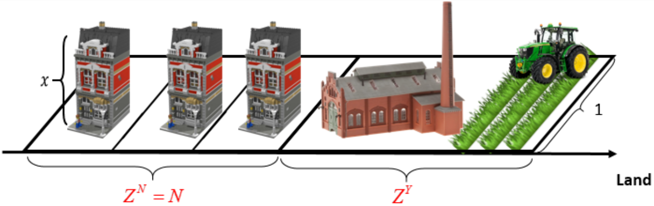



**Numeraire sector**

$$Y = (K)^{\alpha} \left(B^Y L^Y\right)^{\beta} \left(B^Y Z^Y\right)^{1-\alpha-\beta}$$

# Production sectors



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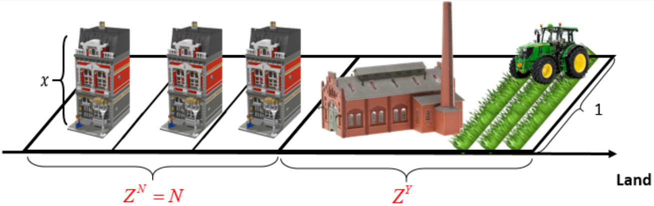


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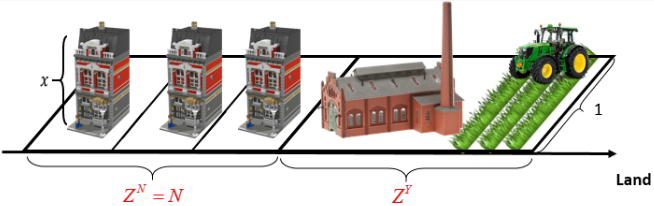
general equilibrium steady state

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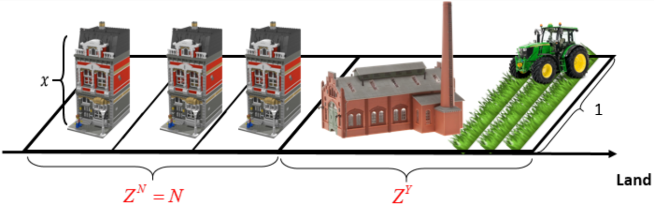
general equilibrium | steady state

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## Market clearing

- Labor: $L^Y + L^X = \sum_j n_j l_j$
- Land: $N + Z^Y \leq Z$

▸ general equilibrium ▸ steady state

# Results: general equilibrium

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  - Policy-reform scenario: $\kappa = 0.17 \to \kappa = 1$

    transitional dynamics towards unconstrained steady state

# Calibration approach

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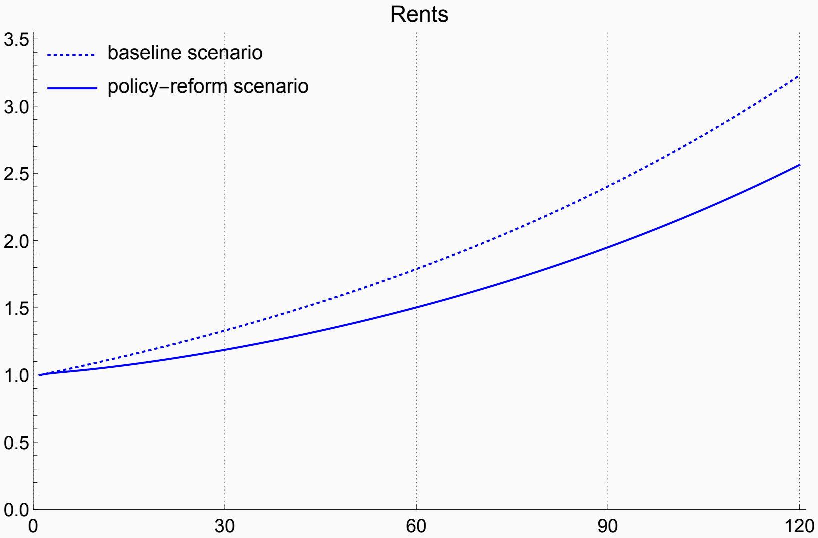
- Housing expenditures

| housing expenditure share in percent | aggregate | income quintile 1st | 2nd | 3rd | 4th | 5th |
|---|---|---|---|---|---|---|
| $\phi = 0$: no status pref. | 19 | 19 | 19 | 19 | 19 | 19 |
| Data: US (2015) | 19 | 25 | 21 | 20 | 19 | 18 |
| $\phi = 0.104$: intermediate status pref. | 19 | 25 | 22 | 20 | 19 | 18 |
| Data: UK (normalized) | 19 | 33 | 23 | 19 | 16 | 15 |
| $\phi = 0.260$: strong status pref. | 19 | 34 | 26 | 23 | 20 | 16 |

▸ Calibration details

# Rent

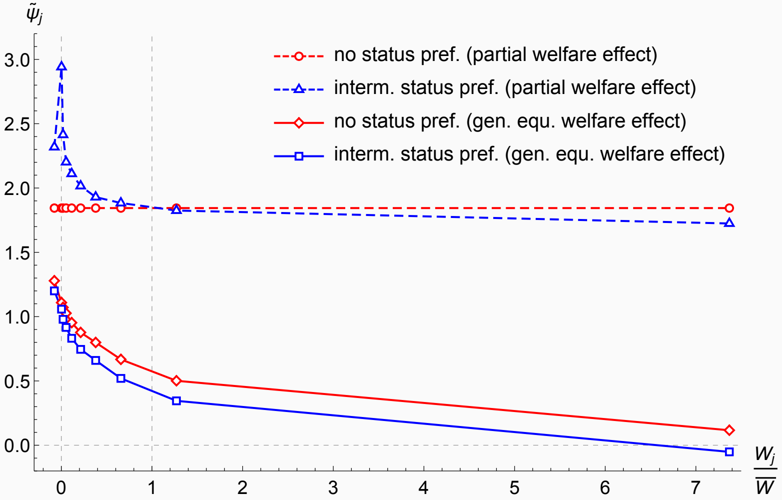


Rents
baseline scenario
policy−reform scenario
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
30
60
90
120

# Wealth inequality


▸ saving rates ▸ Decomposition $\hat{W}_j$

# Welfare: CEV baseline vs policy-reform scenario




details

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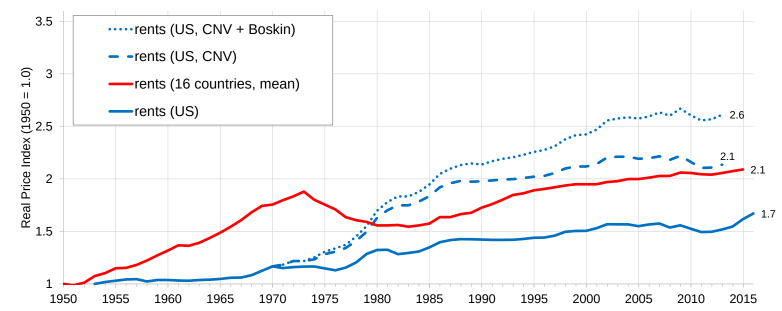
  - Average welfare increases by 0.5 percent
  - Poor benefit more than the rich, richest wealth decile is worse off

# Appendix

# Real rents in the long run

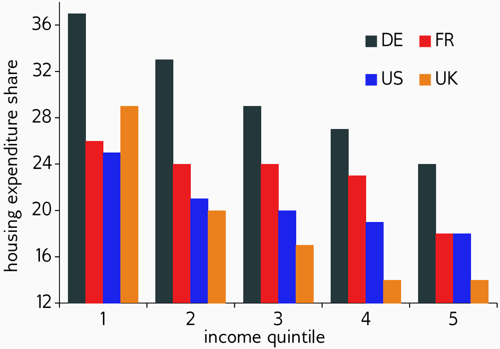



*Source*: US rents: BLS; average rent index: Knoll (2017); revised US rental data: Albouy, Ehrlich, and Liu (2016), based on Crone, Nakamura, and Voith (2010) and the Boskin Comission Report (1996)

- Real rents grow on average between 0.8 and 1.5 percent annually in the US

back

# Schwabe's law




*Source*: US: CEX (2015); UK: ONS (2015); FR: Accardo et al. (2017); DE: Statistisches Bundesamt (2015)

- Historic evidence: Singer (1937, *REStud*), Stigler (1954, *JPE*)
- Recent evidence: Albouy, Ehrlich, & Liu (2016) estimate income elasticity below 1

back

# Alternative interpretations of the term $\phi\bar{s}$

1) **Minimum level of housing consumption** $\phi\bar{s}(t)$, e.g. subsistence, minimum social requirement, physical-, or institutional minimum
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$\rightarrow$ For $\bar{e}(t) = const.$ to hold $\bar{s}(t)$ has to grow at the same rate as aggregate consumption (rising aspirations or changing understanding of poverty)
2) Equivalent formulation: **fixed housing expenditures**

$$u(\tilde{s}_j, c_j) = \frac{\left[\tilde{s}_j(t)^{\theta} c_j(t)^{1-\theta}\right]^{1-\sigma} - 1}{1-\sigma}$$

$$\dot{W}_j(t) + p(t)\tilde{s}_j(t) + c(t) \leq r(t)W_j(t) + w(t)l_j - \underbrace{p(t)\phi\bar{s}(t)}_{\text{fixed housing expenditures}}$$

where $\tilde{s}_j(t) \equiv s_j(t) - \phi\bar{s}(t)$

back

# Alternative formulation of status preferences

- Status preferences are often also captured as ratios instead of differences (Clark et al., 2008, *JEL*):

$$v(s_j, c_j) = \frac{\left[ s_j^{\theta} \left( \frac{s_j}{\bar{s}} \right)^{\phi} c_j^{1-\theta} \right]^{1-\sigma} - 1}{1-\sigma}$$

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- Housing expenditure share:

$$e_j = \frac{\theta + \phi}{1 + \phi}$$

$\Rightarrow v(s_j, c_j)$ cannot capture heterogeneity in housing expenditure shares

back

# Status preferences in both goods

- Generalization of status preferences

$$u(s_j, c_j) = \frac{\left[\left(s_j - \phi_s \bar{s}\right)^{\theta} \left(c_j - \phi_c \bar{c}\right)^{1-\theta}\right]^{1-\sigma} - 1}{1-\sigma}$$

  with $\phi_c, \phi_s \geq 0$ and $\bar{c}$ is the average consumption of the numeraire good
- What matters is the difference $\phi_s - \phi_c$: defining $\phi \equiv \frac{\phi_s - \phi_c}{1-\phi_c}$ yields the same analytical expressions
- **Housing expenditure share declines with income iff** $\phi_s > \phi_c$
  $\rightarrow$ we simplify and set $\phi_c = 0$

back

# Generalization to semi-CES utility

$$u(s_j, c_j) = \frac{\mathcal{C}_j^{1-\sigma} - 1}{1-\sigma}, \quad \text{with} \quad \mathcal{C}_j = \left[\theta\left(s_j - \phi\bar{s}\right)^{1-\frac{1}{\kappa}} + (1-\theta)c_j^{1-\frac{1}{\kappa}}\right]^{\frac{\kappa}{\kappa-1}}$$

- Static elasticity of substitution

$$SES_j = \kappa + \frac{\phi}{\mathfrak{s}_j - \phi}\epsilon_{\mathfrak{s}_j,p}$$

  - For $\phi = 0$ we get $SES_j = \kappa = const.$
  - For $\kappa = 1$ we get $SES_j = 1 + \frac{\phi\bar{s}}{s_j - \phi\bar{s}}\epsilon_{\mathfrak{s}_j,p} < 1$ (Note: $\epsilon_{\mathfrak{s}_j,p} < 0$)
- Housing expenditure share

$$e_j = \frac{\theta^\kappa p^{1-\kappa}}{\theta^\kappa p^{1-\kappa} + (1-\theta)^\kappa\left(1 - \frac{\phi}{\mathfrak{s}_j}\right)}$$

  - $Var(e_j) > 0$ iff $\phi > 0$
  - $\bar{e} = const.$ iff $\kappa = 1$ (Piazzesi and Schneider, 2016)

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# Renters vs. Homeowners

- An economy of homeowners ($s_j = N_j h$)

$$\max_{\{c_j(t), N_j(t)\}_{t=0}^{\infty}} \int_0^{\infty} u\left(c_j(t), N_j(t)h(t)\right) \mathrm{e}^{-\rho t} \mathrm{d}t$$

$$\text{s.t.} \dot{W}_j(t) = r(t)A_j(t) - p^N(t)N_j(t) + w(t)l_j - c_j(t)$$

$$A_j(t) = W_j(t) - P^H(t)N_j(t),$$

- where $p^N \equiv rP^H + \delta^X q^X x + q^X \dot{x} - \dot{P}^H$

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- where $p^N \equiv rP^H + \delta^X q^X x + q^X \dot{x} - \dot{P}^H$
- FOC and all propositions are identical
- Non-arbitrage condition: $ph = p^N \Rightarrow$ Replace $p(t)$ accordingly and *rent channel* becomes a *house price* and *user cost of capital* channel

▸ Rent channel ▸ Welfare

# The dynamics of wealth inequality: analytics

- Growth rate of household-specific wealth

$$\hat{W}_j(t) \equiv \underbrace{sav_j(t)}_{\text{divergence channel}} \underbrace{\frac{r(t)W_j(t) + w(t)l(t)}{W_j(t)}}_{\text{convergence channel}}$$

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# Welfare distribution: baseline scenario

**Consumption-equivalent variation** (Lucas, 1987)

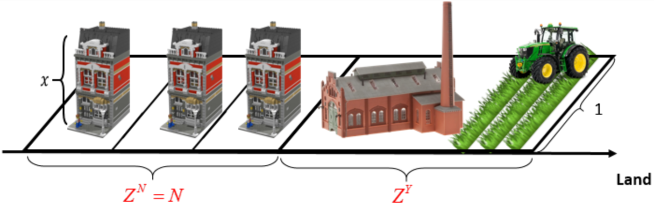
$$\int_0^\infty e^{-\rho t} \frac{\left[\mathcal{C}_j(t)\right]^{1-\sigma} - 1}{1-\sigma} \mathrm{d}t \overset{!}{=} \int_0^\infty e^{-\rho t} \frac{\left[(1+\psi_j)\bar{\mathcal{C}}(t)\right]^{1-\sigma} - 1}{1-\sigma} \mathrm{d}t$$

$\mathcal{C}_j \equiv (s_j - \phi \bar{s})^\theta c_j^{1-\theta}$ and $\bar{\mathcal{C}}$ is average composite consumption

back

# Production sectors



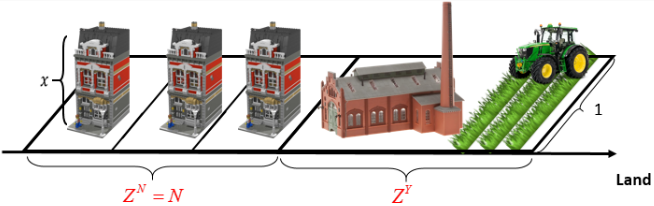



**Numeraire sector**

$$Y = (K)^{\alpha} \left(B^Y L^Y\right)^{\beta} \left(B^Y Z^Y\right)^{1-\alpha-\beta}$$

# Production sectors






## Housing sector

- Housing services supply: $S$

$$S = X^{\gamma} N^{1-\gamma}$$

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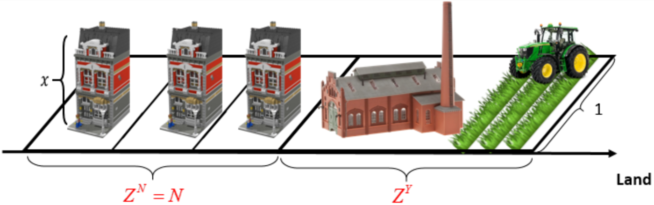


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$$Cost = P^Z \dot{N} + w \frac{\xi}{2} \left(\dot{N}\right)^2, \; N \leq \kappa Z$$

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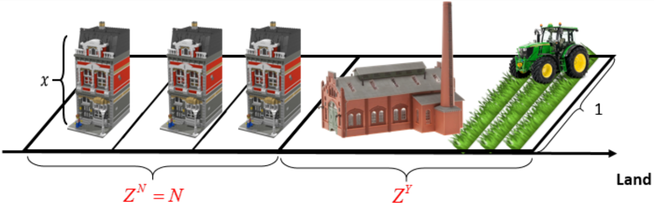
back

# Production sectors



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$$\dot{X} = (M)^{\eta} \left(B^X L^X\right)^{1-\eta} - \delta^X X$$

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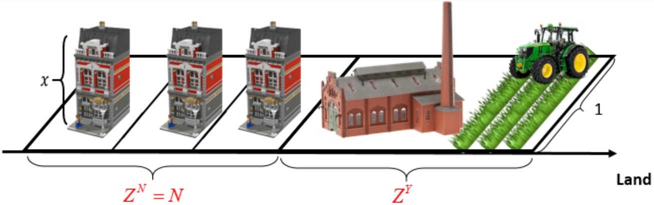
back

# Production sectors



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## Market clearing

- Labor: $L^Y + L^X = \sum_j n_j l_j$
- Land: $N + Z^Y = Z$

back

# Steady state I

## Proposition: Steady state

Assume that $B^Y(t) = B^Y(0)e^{g^Y \cdot t}$ and $B^X(t) = B^X(0)e^{g^X \cdot t}$ with $g^Y, g^X \geq 0$.
The unique steady state growth rates then read as follows

i) Variables $\left\{K, W, C, M, q^N, P^Z, R^Z, P^H, w\right\}$ grow at the rate $g^Y$
ii) Variables $\{X, x\}$ grow at the rate $g^Y + (1-\eta)g^X$
iii) Variable $\{\hat{p}\}$ grow at the rate $(1-\gamma\eta)\, g^Y + \gamma\,(1-\eta)\, g^X$
iv) Variables $\left\{q^X, R^X\right\}$ grow at the rate $(1-\eta)\left(g^Y - g^X\right)$
v) Variables $\{h, S\}$ grow at the rate $\gamma\left(\eta g^Y + (1-\eta)\, g^X\right)$
vi) Variables $\left\{N, Z^Y, L^X, L^Y, r\right\}$ remain constant.

back

# Steady state II

## Proposition: Stationary wealth distribution

i) The steady state wealth distribution is stationary in the sense that, for any two households $j$ and $j'$, the relative wealth position $W_j/W_{j'}$ dœs not change. (Reason: The condition $\mu(t)\tilde{w}(t) = w(t)$ holds in any steady state).

ii) This applies for a zero growth steady state ($g^Y, g^X = 0$) as well as for a positive growth steady state ($g^Y, g^X > 0$).

back

# General equilibrium I

A **general equilibrium** is a sequence of quantities, of prices, and of operating profits of housing services producers

$$\left\{Y(t), K(t), X(t), N(t), M(t), L^Y(t), L^X(t), Z^Y(t)\right\}_{t=0}^{\infty},$$
$$\left\{\left\{c_j(t), s_j(t), W_j(t), K_j(t), Z_j^Y(t), N_j(t)\right\}_{j=1}^{J}\right\}_{t=0}^{\infty},$$
$$\left\{p(t), P^Z(t), q^N(t), q^X(t), w(t), r(t), R^Z(t), R^X(t)\right\}_{t=0}^{\infty}, \{\pi(t)\}_{t=0}^{\infty}$$

for initial distributions $\left\{K_j(0), Z_j^Y(0), N_j(0)\right\}_{j=1}^{J}$ and given $\left\{B^X(t), B^Y(t)\right\}_{t=0}^{\infty}$ such that

i) households maximize lifetime utilities;

ii) representative firms in X sector and Y sector, representative real estate developer, and housing services producers maximize PDV of infinite profit stream, taking prices as given;

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# General equilibrium II

iii) labor markets clear: $L^X(t) + L^Y(t) = L$;

iv) asset markets clear:

$$K(t) = \sum_j \frac{\mathcal{L}}{J} K_j(t),\ N(t) = \sum_j \frac{\mathcal{L}}{J} N_j(t),\ Z^Y(t) = \sum_j \frac{\mathcal{L}}{J} Z_j^Y(t) = Z(t) - N(t);$$

v) perfect arbitrage across all assets holds;

vi) market for housing services clears: $\sum_j \frac{\mathcal{L}}{J} s_j(t) = N(t)h(t)$ ;

vii) market for Y good clears: $Y(t) = C(t) + I^K(t) + I^N(t) + M(t)$ (redundant due to Walras' law).

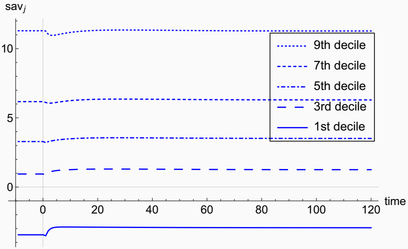
back

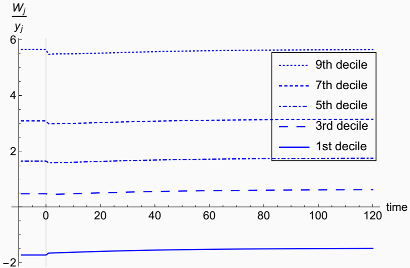
# Calibration

| Parameter | Value | Explanation/Target |
|---|---|---|
| $L$ | 1 | Normalization |
| $J$ | 10 | Match deciles |
| $\{W_j(0)/\bar{W}(0)\}_{j=1}^{J}$ | see text | Wealth deciles (US, SCF, 2013) |
| $\{l_j(0)/\bar{l}(0)\}_{j=1}^{J}$ | see text | average earnings within wealth percentile (US, SCF, 2013) |
| $\sigma$ | 2 | $IES = 0.5$ (Havranek, 2015) |
| $Z$ | 1 | Normalization |
| $\delta^K$ | 0.056 | Davis and Heathcote (2005) |
| $\alpha$ | 0.28 | Land income share in $Y$ sector (Grossmann and Steger, 2017) |
| $\beta$ | 0.69 | Labor expenditure share $Y$ sector (Grossmann and Steger, 2017) |
| $g^Y$ | 0.02 | Growth rate GDP per capita (FRED) |
| $\delta^X$ | 0.015 | Hornstein (2009) |
| $\eta$ | 0.38 | Labor expenditure share $X$ sector (Grossmann and Steger, 2017) |
| $g^X$ | 0.009 | Rent growth: 1% (Knoll, 2017) |
| $\kappa$ | 0.169 | Share of residential land: 16.9 percent (Falcone, 2015) |
| $\theta$ | $\{0.19, \mathbf{0.17}, 0.15\}$ | Average housing expenditure share: 0.19 (CEX, 2015) |
| $\phi$ | $\{0.000, \mathbf{0.104}, 0.260\}$ | Difference between bottom and top income quintiles' housing expenditure share: $\{0, .07, .18\}$ (CEX, 2015; UK) |
| $\rho$ | 0.019 | Real interest rate: 0.0577 (Jorda et al., 2019) |
| $\gamma$ | 0.78 | Land's share in housing wealth: 1/3 |
| $\xi$ | 765 | Transition speed in $N$: 31 percent in 30 years (Davis and Heathcote, 2007) |

▸ back

# Saving rates & wealth-to-income ratios



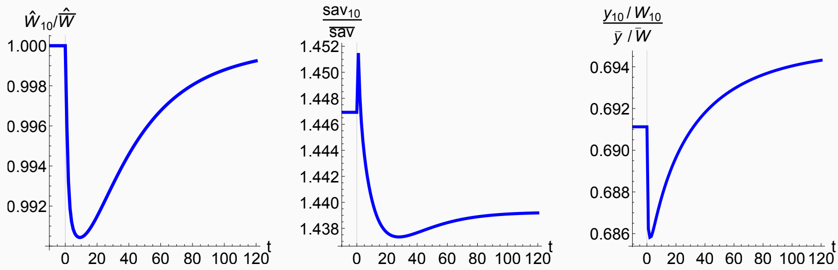




$$\hat{W}_j(t) \equiv sav_j(t) \frac{r(t)W_j(t) + w(t)l}{W_j(t)}$$

$\Rightarrow$ We see that $\frac{\partial sav_j(t)}{\partial W_j(t)} > 0$

back

# Decomposition - counterfactual no zoning experiment




$$\frac{\hat{W}_{10}}{\widehat{\bar{W}}} = \frac{sav_{10}}{s\bar{a}v} \frac{y_{10}/W_{10}}{\bar{y}/\bar{W}}$$

back

# Welfare in general equilibrium: analytics

- Households care about $\{r(t), p(t), w(t)\}_{t=0}^{\infty}$, and $W_j(0)$

back

# Welfare in general equilibrium: analytics

- Households care about $\{r(t), p(t), w(t)\}_{t=0}^{\infty}$, and $W_j(0)$
- Welfare

$$\widetilde{\psi}_j = \left(\frac{\mu^1}{\mu^0}\right)^{\frac{\sigma}{\sigma-1}} \frac{W_j^1 + \tilde{w}^1 l_j}{W_j^0 + \tilde{w}^0 l_j} \left(\frac{\mathcal{P}_j^1}{\mathcal{P}_j^0}\right)^{-1} - 1$$

back

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back

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- Quantitatively relevant channels
    1. $p(t)$ works symmetrically through $\mu$ and asymmetrically (Schwabe's law) through $\mathcal{P}$
       $\rightarrow$ see partial equilibrium plot
       $\rightarrow$ all benefit, total-wealth-poor benefit more (ordering: 2,3,1)

▸ back

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     $\rightarrow$ see partial equilibrium plot
     $\rightarrow$ all benefit, total-wealth-poor benefit more (ordering: 2,3,1)
  2. $W_j(0)$ declines for all in the same proportion
     $\rightarrow$ the higher $W$, the stronger the welfare effect
     $\rightarrow$ non-monotonicity driven by non-monotonicity in $l_j$

back