# Bubble-Driven Business Cycles

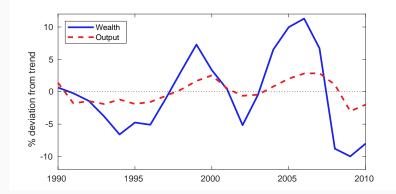
Seminar, University of Bonn

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# Introduction

#### Recent US boom-bust episodes



Notes: Both series are from US NIPA (FRED), expressed in logarithms, deflated with the CPI, and detrended with the HP filter. The smoothing parameter of the HP filter is set to 100 (see, e.g., Backus and Kehoe, 1992, AER or Ríos-Rull, 1996, REStud). Real net wealth in levels: 2001;Q1–2002;Q3: -10.06%, 2007;Q1–2009;Q3: -21.08% Boom-bust episodes have become more prevalent in recent
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- $\rightarrow\,$  This paper: explain boom-bust episodes through the lens of a quantitative macro model with rational bubbles

#### **Research questions**

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- How does an OLG model with rational bubbles and financial frictions perform in a more generalized, quantitative setup?

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- Numerical solution; analyze relevance of different transmission channels; compare model-generated results with observed data

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  - · Bubble explains most of the fluctuations in wealth
  - Robust to including investment adjustment cost



The model

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$$\mathbb{E}\sum_{j=1}^{J}\beta^{j-1}\zeta_{j}u(\boldsymbol{c}_{j},\boldsymbol{l}_{j}), \qquad u(\boldsymbol{c}_{j},\boldsymbol{l}_{j}) = \frac{\left[\boldsymbol{c}_{j}-\boldsymbol{g}^{t}\theta\frac{\boldsymbol{l}_{j}^{l+\chi}}{1+\chi}\right]^{1-\sigma}-1}{1-\sigma}$$

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- Earnings and pensions
  - $j \leq J^{w}$  : supply labor and earn  $(1 \tau)we_{j}l_{j}$
  - $j > J^w$  : retire and receive pen

• A saver becomes an entrepreneur with probability  $p^E$  at age  $J^E$ 

Profit maximization wrt h<sub>i</sub>

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- Entrepreneurs choose the same capital-labor ratios  $\Rightarrow$  aggregation:

$$Y_{t} = Z_{t}^{Y} K_{t}^{\alpha} \left(g^{t} L_{t}\right)^{1-\alpha}$$

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$$R_{t} = 1 + \alpha \frac{Y_{t}}{K_{t}} - \delta$$
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• Entrepreneurs face linear profit function  $\Pi_t(k_{t,j}) = (R_t - 1)k_{t,j}$ 

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- Alternative interpretation: storage technology that transfers 1 unit of income in t into γ units in t + 1
- $\Rightarrow$  Kinked credit supply curve

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- Entrepreneurs face borrowing constraint Kiyotaki and Moore (1997, JPE)

 $R_{t+1}^D d_{t+1,j+1} \leq \mathbb{E}_t W_{t+1,j+1}$ 

$$W_{t+1,j+1} = \underbrace{(1-\delta)k_{t+1,j+1}}_{\text{fundamental}}$$

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$$b_{t+1}^N = g^{t+1} \times v$$

· Aggregate bubble evolves according to

$$B_{t+1} = R_t^B (B_t + B_t^N)$$

$$V^{S}(j,m;\Omega) = \max_{c,l,a'} u(c,l) + \beta \varrho_{j+1} \mathbb{E} V^{S}(j+1,m';\Omega')$$
(3)

subject to

$$c + a' = (1 - \tau)we_j l + m + l_j^r pen + beq$$
  
 $m' = R^{D'}a', a' \ge 0$ 

#### Notes:

- Inferior production vs lending
  - If  ${\it R}^{\it D}=\gamma$  savers are indifferent
  - If  ${\it R}^{\it D} > \gamma$  savers do not use inferior production technology
- Different problem for  $j = J^E 1 \dots$

$$V^{S}(J^{E} - 1, m; \Omega) = \max_{c,l,a'} u(c, l) + p^{E} \beta \varrho_{j+1} \mathbb{E} V^{E}(J^{E}, m'; \Omega')$$

$$+ (1 - p^{E}) \beta \varrho_{j+1} \mathbb{E} V^{S}(J^{E}, m'; \Omega')$$
(4)

subject to

$$c+a'=(1- au)we_{j}l+m+l_{j}^{r}$$
pen+beq $m'=R^{D'}a', \ a'\geq 0$ 

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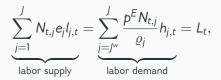
$$c + k' + b' - d' = (1 - \tau) we_{j}l + m + l_{j}^{r} pen + beq$$
$$R^{D'}d' \le (1 - \delta)k' + R^{B'} (b' + b^{N'})$$
$$m' = R'k' - R^{D'}d' + R^{B'} (b' + b^{N'})$$

Note: for  $j = J^E$  net worth is given by  $m = R^D a$ 

# Equilibrium (I of III)

A sequential equilibrium consists of sequences of individual consumption and labor supply  $\{c_{t,j}, l_{t,j}\}_{j=1}^{J}$  for both savers and entrepreneurs as well as of sequences of bubbles, bonds, capital, and debt  $\{b_{t,j}, a_{t,j}, k_{t,j}, d_{t,j}, \}_{j=1}^{J}$  for all  $t \ge 0$  maximizing the household problems eqs. (3) to (5), a sequence of prices  $\{w_t, R_t, R_t^D, R_t^B\}_{t=0}^{\infty}$  satisfying eqs. (1) and (2), a sequence of shocks  $\{Z_t^Y\}_{t=1}^{\infty}$  drawn from its respective distribution and initial values  $\{b_{0,j}, a_{0,j}, k_{0,j}, d_{0,j}\}_{j=1}^{J}, Z_0^Y, R_0^d$  such that

• the labor market clears

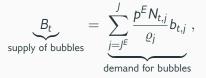


#### Equilibrium (II of III)

• the capital market clears

$$K_t = \sum_{j=J^E+1}^J \frac{p^E N_{t,j}}{\varrho_j} k_{t,j},$$

• the market for bubbles clears



• the credit market clears

$$A_t \equiv \sum_{j=1}^{J^{\mathcal{E}}} \frac{N_{t,j}}{\varrho_j} a_{t,j} + \sum_{s=J^{\mathcal{E}}+1}^J \frac{(1-p^{\mathcal{E}})N_{t,j}}{\varrho_j} a_{t,j} \geq \sum_{j=J^{\mathcal{E}}+1}^J \frac{p^{\mathcal{E}}N_{t,j}}{\varrho_j} d_{t,j} \equiv D_t,$$

## Equilibrium (III of III)

• the government budget is balanced

$$\tau w_t L_t = \sum_{j=J^w+1}^J N_{t,j} pen_t,$$

• the goods market clears

$$Y_t + \gamma(A_t - D_t) = C_t + K_{t+1} - (1 - \delta)K_t + (A_{t+1} - D_{t+1}),$$

· bubbles are freely disposable and the capital stock is positive

 $B_t \geq 0, K_t \geq 0.$ 

Steady state results

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• Testable implications for  $gn, R^D, R$  in postwar US  $_{Jorda \ et \ al. \ (2019, \ QE)}$ 

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- With  $\gamma > 0$ : similar conclusions, except that  $R^D > \gamma$ 
  - If  $R^D > \gamma$ : Savers do not use inferior technology
  - If  $R^D = \gamma$ : Savers use inferior technology and  $R^D$  constant

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#### Calibration strategy postwar US; annual: $\gamma = 0$

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  - match R = 1.067 by setting  $\delta = 0.052$  ( $\frac{K}{Y}$  will be matched with  $\beta$ )

$$\delta = 1 + \alpha \left( K/Y \right)^{-1} - R.$$

 $\Rightarrow$  implied investment-output ratio of 0.23 (0.174 in the data)



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  - setting g = 1.02 and n = 1.011 as observed in the data
  - match R = 1.067 by setting  $\delta = 0.052$  ( $\frac{K}{Y}$  will be matched with  $\beta$ )

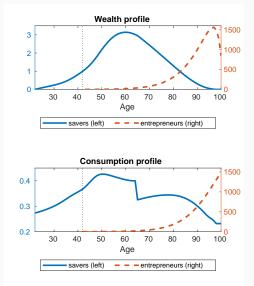
$$\delta = 1 + \alpha \left( K/Y \right)^{-1} - R.$$

 $\Rightarrow$  implied investment-output ratio of 0.23 (0.174 in the data)

• match  $R - R^D = 0.05$  by setting  $\eta = 0.001$  ( $p^E = 0.002$ )



#### Entrepreneurs and savers experience very different lives



 $\rightarrow$  CEV of entrepreneur's vs saver's life: 232 %

| General eq            | uil. |                | General equil. |
|-----------------------|------|----------------|----------------|
| fundamental           |      | Partial equil. | bubbly         |
|                       |      |                |                |
| Output Y              | 0    |                | 0.9            |
| Capital K             | 0    |                | 2.2            |
| Labor <i>L</i>        | 0    |                | 0.2            |
| Credit demand D       | 0    |                | 2.2            |
| Savers' wealth A      | 0    |                | 2.2            |
| Entrep.' wealth $A^E$ | 0    |                | 3.2            |
| Bubble B              | 0    |                | 1              |

All in %, bubble relative to wealth, other relative to previous column. Prices:  $b^N \uparrow$ ,  $w \uparrow$ , pen  $\uparrow$ , beq  $\uparrow$ , R  $\downarrow$ , R<sup>D</sup>  $\uparrow$ 

 $\rightarrow$  Total effect of bubbles: expansionary

| General equil.        |   |      |                | General equil. |  |  |
|-----------------------|---|------|----------------|----------------|--|--|
| fundamental           |   |      | Partial equil. | bubbly         |  |  |
|                       |   | ЬN   |                |                |  |  |
| Output Y              | 0 | 7.6  |                |                |  |  |
| Capital K             | 0 | 24.4 |                |                |  |  |
| Labor <i>L</i>        | 0 | 0    |                |                |  |  |
| Credit demand D       | 0 | 24.4 |                |                |  |  |
| Savers' wealth A      | 0 | -0.0 |                |                |  |  |
| Entrep.' wealth $A^E$ | 0 | 24.4 |                |                |  |  |
| Bubble B              | 0 | 0    |                |                |  |  |

All in %, bubble relative to wealth, other relative to previous column. Prices:  $b^{N} \uparrow$ ,  $w \uparrow$ , pen  $\uparrow$ , beq  $\uparrow$ ,  $R \downarrow$ ,  $R^{D} \uparrow$ 

 $\rightarrow$  Bubble-creation channel: expansionary (if borrowing constraint binding)  $$Martin and Ventura (2012 AER, 2016 JEEA)$}$ 

$$R^{D}d \leq (1-\delta)k + R^{B}(b+b^{N})$$

| General equil.        |     |       | General equil. |        |  |
|-----------------------|-----|-------|----------------|--------|--|
| fundamen              | tal |       | Partia         | bubbly |  |
|                       |     | $b^N$ | w + pen + beq  |        |  |
| Output Y              | 0   | 7.6   | 1.8            |        |  |
| Capital K             | 0   | 24.4  | 0.8            |        |  |
| Labor <i>L</i>        | 0   | 0     | 2.3            |        |  |
| Credit demand D       | 0   | 24.4  | 0.8            |        |  |
| Savers' wealth A      | 0   | -0.0  | 0.8            |        |  |
| Entrep.' wealth $A^E$ | 0   | 24.4  | 0.8            |        |  |
| Bubble B              | 0   | 0     | 0              |        |  |

All in %, bubble relative to wealth, other relative to previous column. Prices:  $b^N \uparrow$ ,  $w \uparrow$ , pen  $\uparrow$ , beq  $\uparrow$ , R  $\downarrow$ , R<sup>D</sup>  $\uparrow$ 

 $\rightarrow$  Amplified by higher wages and pensions

| General equil.<br>fundamental |     |                | General equil.<br>bubbly |          |       |  |
|-------------------------------|-----|----------------|--------------------------|----------|-------|--|
| Tunuamen                      | Lai | -              | 10                       | artial e | 4uii. |  |
|                               |     | Ь <sup>N</sup> | w + pen + beq            | R        |       |  |
| Output Y                      | 0   | 7.6            | 1.8                      | -4.9     |       |  |
| Capital K                     | 0   | 24.4           | 0.8                      | -13.5    |       |  |
| Labor <i>L</i>                | 0   | 0              | 2.3                      | -1.2     |       |  |
| Credit demand D               | 0   | 24.4           | 0.8                      | -13.5    |       |  |
| Savers' wealth A              | 0   | -0.0           | 0.8                      | 0        |       |  |
| Entrep.' wealth $A^E$         | 0   | 24.4           | 0.8                      | -13.5    |       |  |
| Bubble B                      | 0   | 0              | 0                        | 0        |       |  |

All in %, bubble relative to wealth, other relative to previous column. Prices:  $b^N \uparrow$ ,  $w \uparrow$ , pen  $\uparrow$ , beq  $\uparrow$ , R  $\downarrow$ , R<sup>D</sup>  $\uparrow$ 

 $\rightarrow$  Dampened by lower return on capital

| General equil.<br>fundamental |   |                | Pa            | General equil.<br>bubbly |                        |  |
|-------------------------------|---|----------------|---------------|--------------------------|------------------------|--|
|                               |   |                |               |                          |                        |  |
|                               |   | Ь <sup>N</sup> | w + pen + beq | R                        | savers' R <sup>D</sup> |  |
| Output Y                      | 0 | 7.6            | 1.8           | -4.9                     | 0.3                    |  |
| Capital K                     | 0 | 24.4           | 0.8           | -13.5                    | 0.7                    |  |
| Labor <i>L</i>                | 0 | 0              | 2.3           | -1.2                     | 0.1                    |  |
| Credit demand D               | 0 | 24.4           | 0.8           | -13.5                    | 0.7                    |  |
| Savers' wealth A              | 0 | -0.0           | 0.8           | 0                        | 1.4                    |  |
| Entrep.' wealth $A^E$         | 0 | 24.4           | 0.8           | -13.5                    | 0.7                    |  |
| Bubble B                      | 0 | 0              | 0             | 0                        | 0                      |  |

All in %, bubble relative to wealth, other relative to previous column. Prices:  $b^N \uparrow$ ,  $w \uparrow$ , pen  $\uparrow$ , beq  $\uparrow$ , R  $\downarrow$ , R<sup>D</sup>  $\uparrow$ 

-> Liquidity channel: expansionary (if borrowing constraint binding)

Farhi and Tirole (2012 REStud)

Higher  $R^D \Rightarrow$  savers save more  $\Rightarrow$  new entrepreneurs start with higher net worth  $\Rightarrow$  more investment

| General equil.<br>fundamental |   |                | General equil.<br>bubbly |       |                        |                         |  |
|-------------------------------|---|----------------|--------------------------|-------|------------------------|-------------------------|--|
|                               |   | Ь <sup>N</sup> | w + pen + beq            | R     | savers' R <sup>D</sup> | entrep.' R <sup>D</sup> |  |
| Output Y                      | 0 | 7.6            | 1.8                      | -4.9  | 0.3                    | -3.9                    |  |
| Capital K                     | 0 | 24.4           | 0.8                      | -13.5 | 0.7                    | -10.2                   |  |
| Labor <i>L</i>                | 0 | 0              | 2.3                      | -1.2  | 0.1                    | -1                      |  |
| Credit demand D               | 0 | 24.4           | 0.8                      | -13.5 | 0.7                    | -10.2                   |  |
| Savers' wealth A              | 0 | -0.0           | 0.8                      | 0     | 1.4                    | 0                       |  |
| Entrep.' wealth $A^E$         | 0 | 24.4           | 0.8                      | -13.5 | 0.7                    | -9.2                    |  |
| Bubble <i>B</i>               | 0 | 0              | 0                        | 0     | 0                      | 1                       |  |

All in %, bubble relative to wealth, other relative to previous column. Prices:  $b^{N} \uparrow$ ,  $w \uparrow$ , pen  $\uparrow$ , beq  $\uparrow$ ,  $R \downarrow$ ,  $R^{D} \uparrow$ 

 $\rightarrow$  crowding-out channel: always contractionary

Tirole (1985 Ectra)

$$R^{D}d \leq (1-\delta)k + R^{B}(b+b^{N})$$

Higher  $R^D \Rightarrow$  Lower leverage: bubbles crowd-out capital

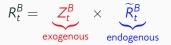
| General equil.<br>fundamental |   |      | Pa            | General equil.<br>bubbly |                        |                         |     |
|-------------------------------|---|------|---------------|--------------------------|------------------------|-------------------------|-----|
|                               |   | Ь    | w + pen + beq | R                        | savers' R <sup>D</sup> | entrep.' R <sup>D</sup> |     |
| Output Y                      | 0 | 7.6  | 1.8           | -4.9                     | 0.3                    | -3.9                    | 0.9 |
| Capital K                     | 0 | 24.4 | 0.8           | -13.5                    | 0.7                    | -10.2                   | 2.2 |
| Labor <i>L</i>                | 0 | 0    | 2.3           | -1.2                     | 0.1                    | -1                      | 0.2 |
| Credit demand D               | 0 | 24.4 | 0.8           | -13.5                    | 0.7                    | -10.2                   | 2.2 |
| Savers' wealth A              | 0 | -0.0 | 0.8           | 0                        | 1.4                    | 0                       | 2.2 |
| Entrep.' wealth $A^E$         | 0 | 24.4 | 0.8           | -13.5                    | 0.7                    | -9.2                    | 3.2 |
| Bubble B                      | 0 | 0    | 0             | 0                        | 0                      | 1                       | 1   |

All in %, bubble relative to wealth, other relative to previous column. Prices: *b<sup>N</sup>* ↑, *w* ↑, *pen* ↑, *beq* ↑, *R* ↓, *R<sup>D</sup>* ↑

- · Bubbles have an expansionary effect in the calibrated model
- Mainly through the bubble-creation channel
- Liquidity channel almost irrelevant

Bubble-driven business cycles

• Stochastic bubbles: ex-post return on bubbles



• Market sentiment:  $\ln Z_t^B$  is a Gaussian white noise with variance  $\sigma^B$ 

• Stochastic bubbles: ex-post return on bubbles

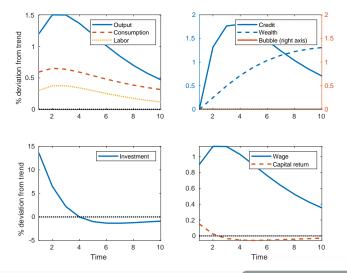


- Market sentiment:  $\ln Z_t^B$  is a Gaussian white noise with variance  $\sigma^B$
- EGM for solving household problem and IRF as a numerical derivative obtained from MIT shocks Boppart, Krusell and Mitman (2018, JEDC)
- Focus on  $\gamma = \bar{\mathbf{R}}^{\mathbf{D}} + \epsilon$ , with small  $\epsilon$

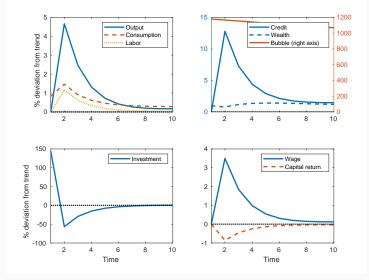




#### A real business cycle (1% TFP shock)



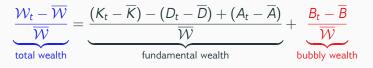
differences to Ríos-Rull, 1996, REStud



Two recent US boom-bust episodes

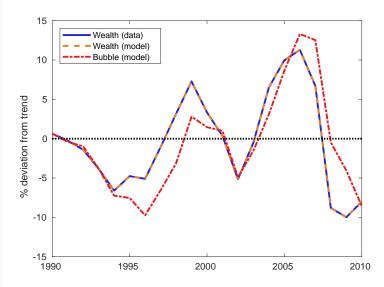
• 1990–2010: match  $\{Y_t, \mathcal{W}_t\}_{t=1990}^{2010}$  by solving linear IRFs for innovations  $\{\epsilon_t^{\rm Y}, \epsilon_t^{\rm B}\}_{t=1990}^{2010}$ 

- 1990–2010: match  $\{Y_t, W_t\}_{t=1990}^{2010}$  by solving linear IRFs for innovations  $\{\epsilon_t^Y, \epsilon_t^B\}_{t=1990}^{2010}$
- Decomposition

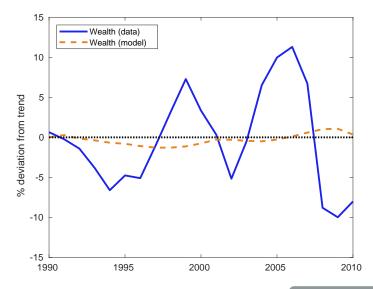


where  $W_t \equiv K_t - D_t + A_t + B_t$ .

#### Recent US boom-bust episodes were driven by a bubble ...



#### ... and cannot be explained by TFP shocks alone



• Explain boom-bust episodes trough the lens of a quantitative overlapping-generations RBC model (Rios-Rull, 1996 *REStud*) with rational, stochastic bubbles and financial frictions

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- · Calibrated model can generate expansionary bubbles

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- Bubbles affect macroeconomy through different channels

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- Calibrated model can generate expansionary bubbles
- Bubbles affect macroeconomy through different channels
  - Bubble-creation channel necessary for expansionary bubbles to
     exist
     Martin and Ventura (2012 AER, 2016 JEEA)

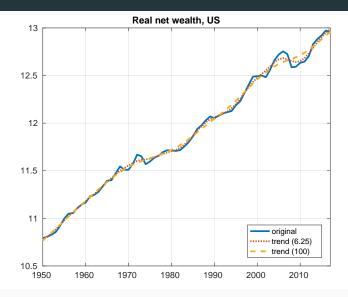
- Explain boom-bust episodes trough the lens of a quantitative overlapping-generations RBC model (Rios-Rull, 1996 *REStud*) with rational, stochastic bubbles and financial frictions
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  - Liquidity channel quantitatively negligible Farhi and Tirole (2012 ECTA)

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  - New channel in multi-period setting: stochastic channel of bubbles

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  - New channel in multi-period setting: stochastic channel of bubbles
- Recent US boom-bust episodes can be explained by *bubble-driven* business cycle

# Appendix

#### HP-filtered US wealth



## Literature: quant DSGE literature on boom-bust episodes

• Amplification of fundamental shocks through financial sector

Bernanke and Gertler (1989 AER); Kiyotaki and Moore (1997 JPE); Christiano et al. (2015 AEJ:M); ...

- Shocks emanating in financial sector, e.g. "valuation shocks" and "liquidity shocks" Gertler and Karadi (2011 JME): Kiyotaki and Moore (2012 NBER): ....
- · Shocks to utility function parameters

lacoviello (2005 AER); lacoviello and Neri (2010 AEJ:M); Kaplan at al. (forth. JPE); ...

• Exogenous or near-rational bubbles

Bernanke and Gertler (1999, 2001), Luik and Wesselbaum (2014 JM); Adam et al. (2017 AER); ...

 $\rightarrow$  Explain boom-bust episodes with **rational bubbles** 

### Literature: rational bubbles in GE models

- Early models: contractionary bubbles Samuelson (1958 JPE); Tirole (1985 Ectra)
- Recent models: expansionary bubbles by adding (financial) frictions

Farhi and Tirole (2012 Ectro); Martin and Ventura (2012 AER, 2016 JEEA, 2018 ARE); Galí (2014 AER); ...

- $\rightarrow\,$  large-scale OLG, concave utility, TFP shocks, endogenous labor supply, ...
- ightarrow confront theory with the data
- More quantitative Miao, Wang and Xu (2015, *QE*); Galí (2018); Domeij and Ellingson (2018, *JME*)
  - $\rightarrow$  Different mechanisms, very small effects
- → First to consider rational bubbles within a DSGE model with overlapping generations and financial frictions



# Entrepreneurs: static labor choice (I)

• Static problem ( $k_{t,j}$  and  $Z_t^{\gamma}$  are given in t)

$$\max_{h_{t,j}} \Pi_{t,j} = Z_t^{\gamma} k_{t,j}^{\alpha} \left( g^t h_{t,j} \right)^{1-\alpha} - w_t h_{t,j} - \delta k_{t,j},$$

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#### Entrepreneurs: static labor choice (I)

- Static problem ( $k_{t,j}$  and  $Z_t^{Y}$  are given in t)  $\max_{h_{t,j}} \prod_{t,j} = Z_t^{Y} k_{t,j}^{\alpha} (g^t h_{t,j})^{1-\alpha} - w_t h_{t,j} - \delta k_{t,j},$
- FOC

$$(1-\alpha)Z_t^{Y}\left(g^t\right)^{1-\alpha}\left(\frac{k_{t,j}}{h_{t,j}}\right)^{\alpha} = w_t$$

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- Static problem  $(k_{t,j} \text{ and } Z_t^Y \text{ are given in } t)$  $\max_{h_{t,j}} \prod_{t,j} = Z_t^Y k_{t,j}^\alpha \left(g^t h_{t,j}\right)^{1-\alpha} - w_t h_{t,j} - \delta k_{t,j},$
- FOC

$$(1-\alpha)Z_t^{Y}\left(g^t\right)^{1-\alpha}\left(\frac{k_{t,j}}{h_{t,j}}\right)^{\alpha} = w_t$$

• Same capital-labor ratios  $\Rightarrow$  aggregation:

$$w_t = (1 - \alpha) Z_t^{Y} \left( g^t \right)^{1 - \alpha} \left( \frac{K_t}{L_t} \right)^{\alpha}$$

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- Static problem ( $k_{t,j}$  and  $Z_t^Y$  are given in t)  $\max_{h_{t,j}} \prod_{t,j} = Z_t^Y k_{t,j}^{\alpha} \left(g^t h_{t,j}\right)^{1-\alpha} - w_t h_{t,j} - \delta k_{t,j},$
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$$(1-\alpha)Z_t^{\mathsf{Y}}\left(g^t\right)^{1-\alpha}\left(\frac{k_{t,j}}{h_{t,j}}\right)^{\alpha} = w_t$$

• Same capital-labor ratios  $\Rightarrow$  aggregation:

$$w_t = (1 - \alpha) Z_t^{Y} \left( g^t \right)^{1 - \alpha} \left( \frac{K_t}{L_t} \right)^{\alpha}$$

Aggregate output

$$y_{t,j} = Z_t^Y (g^t)^{1-\alpha} \left(\frac{k_{t,j}}{h_{t,j}}\right)^{\alpha-1} k_{t,j} = Z_t^Y (g^t)^{1-\alpha} \left(\frac{K}{L}\right)^{\alpha-1} k_{t,j}$$
$$\Rightarrow Y_t = Z_t^Y K_t^\alpha (g^t L_t)^{1-\alpha}$$

# Entrepreneurs: static labor choice (II)

• From FOC

$$w_{t} = (1 - \alpha) Z_{t}^{Y} \left(g^{t}\right)^{1 - \alpha} \left(\frac{K_{t}}{L_{t}}\right)^{\alpha}$$
$$h_{t,j} = \frac{L_{t}}{K_{t}} k_{t,j}$$
$$\Rightarrow w_{t} h_{t,j} = (1 - \alpha) Z_{t}^{Y} \left(\frac{K_{t}}{g^{t} L_{t}}\right) k_{t,j}$$



## Entrepreneurs: static labor choice (II)

• From FOC

$$w_{t} = (1 - \alpha) Z_{t}^{Y} \left(g^{t}\right)^{1 - \alpha} \left(\frac{K_{t}}{L_{t}}\right)^{\alpha}$$
$$h_{t,j} = \frac{L_{t}}{K_{t}} k_{t,j}$$
$$\Rightarrow w_{t} h_{t,j} = (1 - \alpha) Z_{t}^{Y} \left(\frac{K_{t}}{g^{t} L_{t}}\right) k_{t,j}$$

• Profit function

$$\Pi_{t}(k_{t,s}) = Z_{t}^{Y} \left(\frac{k_{t,j}}{g^{t}h_{t,j}}\right)^{\alpha-1} k_{t,j} - w_{t}h_{t,j} - \delta k_{t,j}$$
$$= Z_{t}^{Y} \left(\frac{K_{t}}{g^{t}L_{t}}\right)^{\alpha-1} k_{t,j} - (1-\alpha)Z_{t}^{Y} \left(\frac{K}{g^{t}L}\right) k_{t,j} - \delta k_{t,j}$$
$$= \left[\alpha Z_{t}^{Y} \left(\frac{g^{t}L_{t}}{K_{t}}\right)^{1-\alpha} - \delta\right] k_{t,j} = (R_{t}-1)k_{t,j}$$

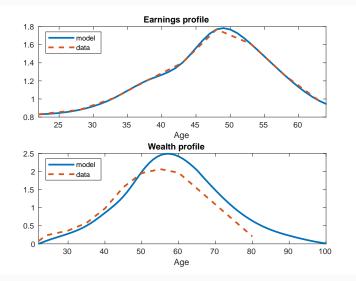
## Externally calibrated parameters

| Parameter                    |                         | Value        | Explanation/Target                                       |
|------------------------------|-------------------------|--------------|--|
| Life span                    | J                       | 79           | life spane of 85 years                                   |
| Period of entrepr. shock     | $J^E$                   | 21           | mean age of founding entrepr.: 42 (Azoulay et al., 2018) |
| Retirement                   | J <sup>w</sup>          | 43           | retirement at 65   |
| Population growth            | n                       | 1.011        | UN (2017)  |
| Replacement ratio            | ξ                       | 0.5          | Imrohoroglu et al. (1995 <i>ET</i> )                     |
| Inverse of IES               | $\sigma$                | 4            | Havranek (2015 <i>JEEA</i> )                             |
| Inverse of Frisch elasticity | χ                       | 3            | standard value   |
| Productivity profile         | $\{e_{j}\}_{j=1}^{J}$   | see text     | Earnings profile in 1995 (SCF)                           |
| Survival probabilities       | $\{\zeta_j\}_{j=1}^{j}$ | see text     | Anderson (1999)  |
| Capital income share         | α                       | 1/3          | standard value   |
| Technological growth         | g                       | 1.02         | per-capita GDP growth                                    |
| Depreciation                 | δ                       | 0.052        | Return on capital 6.7%                                   |
| TFP shock autocorr.          | $\rho^{Y}$              | 0.814        | Prescott (1986)  |
| TFP shock volatility         | $\sigma^{Y}$            | 0.014        | Prescott (1986)  |
| Bubble shock volatility      | $\sigma^{\rm B}$        | $\sigma^{Y}$ | baseline   |

| Parameter             |         | Value  | Target                                   | Value         |
|-----------------------|---------|--------|--|---------------|
| Disutility from labor | θ       | 31.105 | time spent working                       | $\frac{1}{3}$ |
| Discount factor       | $\beta$ | 1.116  | capital output ratio (BEA)               | 2.8           |
| Share entrepreneurs   | $\eta$  | 0.001  | return differential (Jorda et al., 2018) | 0.05          |
| Bubble creation       | ν       | 0.449  | entrepreneurial wealth share of bubble   | 0.01          |



## Cross-sectional fit



| General eq<br>fundamen |   |                | Pa            | artial equil. |                        |                         | General equil.<br>bubbly |
|------------------------|---|----------------|---------------|---------------|------------------------|-------------------------|--------------------------|
|                        |   | b <sup>N</sup> | w + pen + beq | R             | savers' R <sup>d</sup> | entrep.' R <sup>d</sup> |                          |
| Output Y               | 0 | 7.6 (7.6)      | 0.3 (1.8)     | -4.4 (-4.9)   | 0.3 (0.3)              | -3.7 (-3.9)             | 0.9                      |
| Capital K              | 0 | 24.4 (24.4)    | 0.8 (0.8)     | -10.7 (-13.5) | 0.8 (0.7)              | -9 (-10.2)              | 2.2                      |
| Labor <i>L</i>         | 0 | 0 (0)          | 0.1 (2.3)     | -1.1 (-1.2)   | 0.1 (0.1)              | -0.9 (-1)               | 0.2                      |
| Credit demand D        | 0 | 24.4 (24.4)    | 0.8 (0.8)     | -10.7 (-13.5) | 0.8 (0.7)              | -9 (-10.2)              | 2.2                      |
| Savers' wealth A       | 0 | -0.0 (-0.0)    | 0.8 (0.8)     | 0 (0)         | 1.4 (1.4)              | 0 (0)                   | 2.2                      |
| Entrep.' wealth $A^E$  | 0 | 24.4 (24.4)    | 0.8 (0.8)     | -10.7 (-13.5) | 0.8 (0.7)              | -8.1 (-9.2)             | 3.2                      |
| Bubble B               | 0 | 0 (0)          | 0 (0)         | 0 (0)         | 0 (0)                  | 1 (1)                   | 1                        |

All in %, bubble relative to wealth, other relative to fundamental state. Prices:  $b^N$   $\uparrow$ , w  $\uparrow$ , pen  $\uparrow$ , beq  $\uparrow$ , R  $\downarrow$ , R<sup>d</sup>  $\uparrow$ 

- Different ordering: not cumulative, but isolated comparison to fundamental equilibrium
- Very similar results

| General eq<br>fundamen |   | Partial equil. |               |               |                        |                         | General equil.<br>bubbly |
|------------------------|---|----------------|---------------|---------------|------------------------|-------------------------|--------------------------|
|                        |   | ЬN             | w + pen + beq | R             | savers' R <sup>d</sup> | entrep.' R <sup>d</sup> |                          |
| Output Y               | 0 | 7.4 (7.6)      | 2 (1.8)       | -8 (-4.9)     | 0 (0.3)                | 0 (-3.9)                | 1.4 (0.9)                |
| Capital K              | 0 | 24 (24.4)      | 1.2 (0.8)     | -21.6 (-13.5) | 0 (0.7)                | 0 (-10.2)               | 3.6 (2.2)                |
| Labor <i>L</i>         | 0 | 0 (0)          | 2.3 (2.3)     | -1.9 (-1.2)   | 0 (0.1)                | 0 (-1)                  | 0.4 (0.2)                |
| Credit demand D        | 0 | 24 (24.4)      | 1.2 (0.8)     | -17 (-13.5)   | 0 (0.7)                | 0.1 (-10.2)             | 3.7 (2.2)                |
| Savers' wealth A       | 0 | -0.0 (-0.0)    | 1.3 (0.8)     | 0 (0)         | 0 (1.4)                | 0 (0)                   | 1.3 (2.2)                |
| Entrep.' wealth $A^E$  | 0 | 24 (24.4)      | 1.2 (0.8)     | -17 (-13.5)   | 0 (0.7)                | 0 (-9.2)                | 3.6 (3.2)                |
| Bubble B               | 0 | 0 (0)          | 0 (0)         | 0 (0)         | 0 (0)                  | 1.3 (1)                 | 1.3                      |

All in %, bubble relative to wealth, other relative to previous column. Prices:  $b^N \uparrow$ ,  $w \uparrow$ , pen  $\uparrow$ , beq  $\uparrow$ ,  $R \downarrow$ 

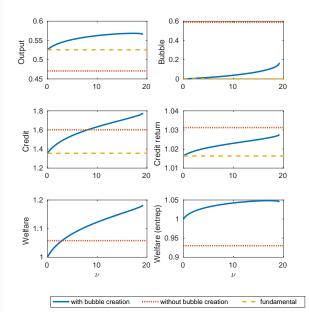
- No liquidity channel
- No crowding-out channel
- · Only expansionary bubble-creation channel

# Decomposition with $\gamma = R^d$

|                      | General equilibrium |                                | Partial equili        | General equilibrium |                                |                                       |
|----------------------|---------------------|--------------------------------|-----------------------|---------------------|--------------------------------|---------------------------------------|
|                      | without bubbles     |                                |                       |                     |                                | with bubbles                          |
|                      | (1)                 | (2)<br>+ <i>b</i> <sup>N</sup> | (3)<br>+w + pen + beq | (4)<br>+R           | (5)<br>+savers' R <sup>d</sup> | (6)<br>+entrepreneurs' R <sup>d</sup> |
| Output Y             | 0.0                 | 7.5                            | 9.5                   | 1.4                 | 1.4                            | 1.4                                   |
| Capital K            | 0.0                 | 24.1                           | 25.4                  | 3.7                 | 3.7                            | 3.7                                   |
| Labor <i>L</i>       | 0.0                 | 0.0                            | 2.3                   | 0.4                 | 0.4                            | 0.4                                   |
| Credit D             | 0.0                 | 24.1                           | 25.4                  | 3.7                 | 3.7                            | 3.7                                   |
| Savers' wealth A     | 0.0                 | -0.001                         | 1.3                   | 1.3                 | 1.3                            | 1.3                                   |
| Entrep' wealth $A^E$ | 0.0                 | 24.1                           | 25.4                  | 3.7                 | 3.7                            | 3.7                                   |
| Bubble B             | 0.0                 | 0.0                            | 0.0                   | 0.0                 | 0.0                            | 1.3                                   |

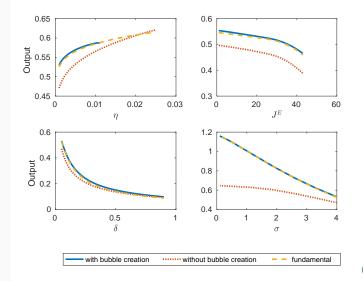
- No liquidity channel
- No crowding-out channel
- Only expansionary bubble-creation channel

#### Robustness: $\nu$



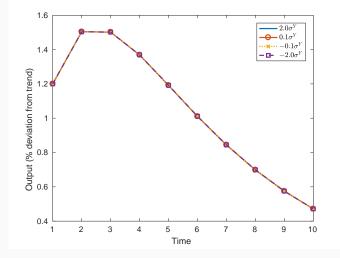
back

# Robustness: $\eta, J^{E}, \delta, \sigma$



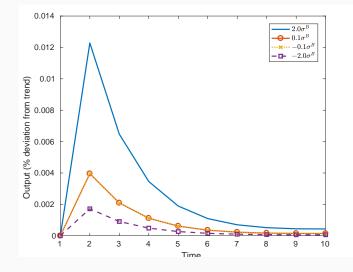
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## Is the IRF of GDP linear in TFP-shocks?



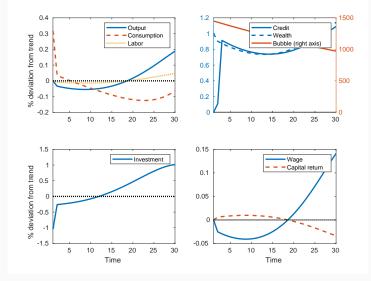
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#### Is the IRF of GDP linear in bubble-shocks?



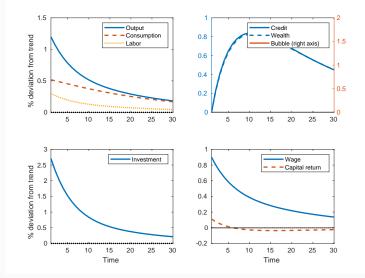
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#### A bubble-driven business cycle under $\gamma = 0$

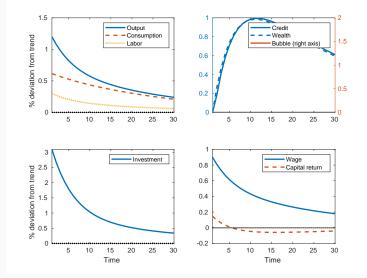


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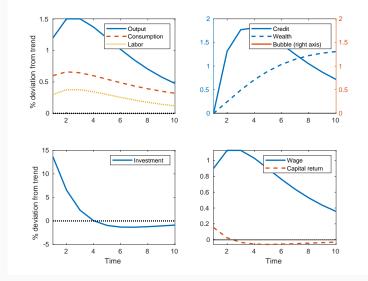
### *Real* business cycle with $\gamma = 0$ , no fin. frict., no bubble



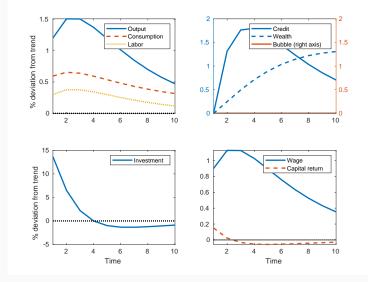
### Real business cycle: + financial frictions

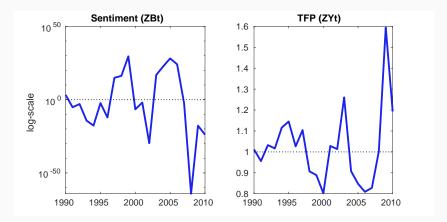


### *Real* business cycle: + $\gamma >> 0$



#### Real business cycle: + bubbles





▶ back

### Investment adjustment cost

• Infinitely-lived, mass-zero, risk-neutral firm sector produces capital under perfect competition

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ q_t l_t \left[ 1 - \frac{\psi}{2} \left( \frac{l_t}{l_{t-1}} - gn \right)^2 \right] - l_t \right\}.$$

• FOC yields

$$\left[1-\frac{\psi}{2}\left(\frac{l_t}{l_{t-1}}-gn\right)\left(3\frac{l_t}{l_{t-1}}-gn\right)\right]q_t=1+\beta\mathbb{E}_t\phi\left(\frac{l_{t+1}}{l_t}-gn\right)\left(\frac{l_{t+1}}{l_t}\right)^2q_{t+1}.$$

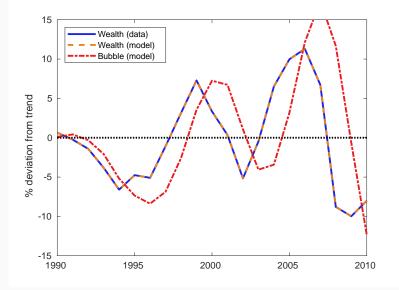
[PRELIMINARY]

· Law of motion of capital

$$\mathcal{K}_{t+1} = (1-\delta)\mathcal{K}_t + I_t \left[1 - \frac{\phi}{2}\left(\frac{I_t}{I_{t-1}} - gn\right)^2\right]$$

## Investment adjustment cost: US bubble

## [PRELIMINARY]



## Investment adjustment cost: without bubbles [PRELIMINARY]

